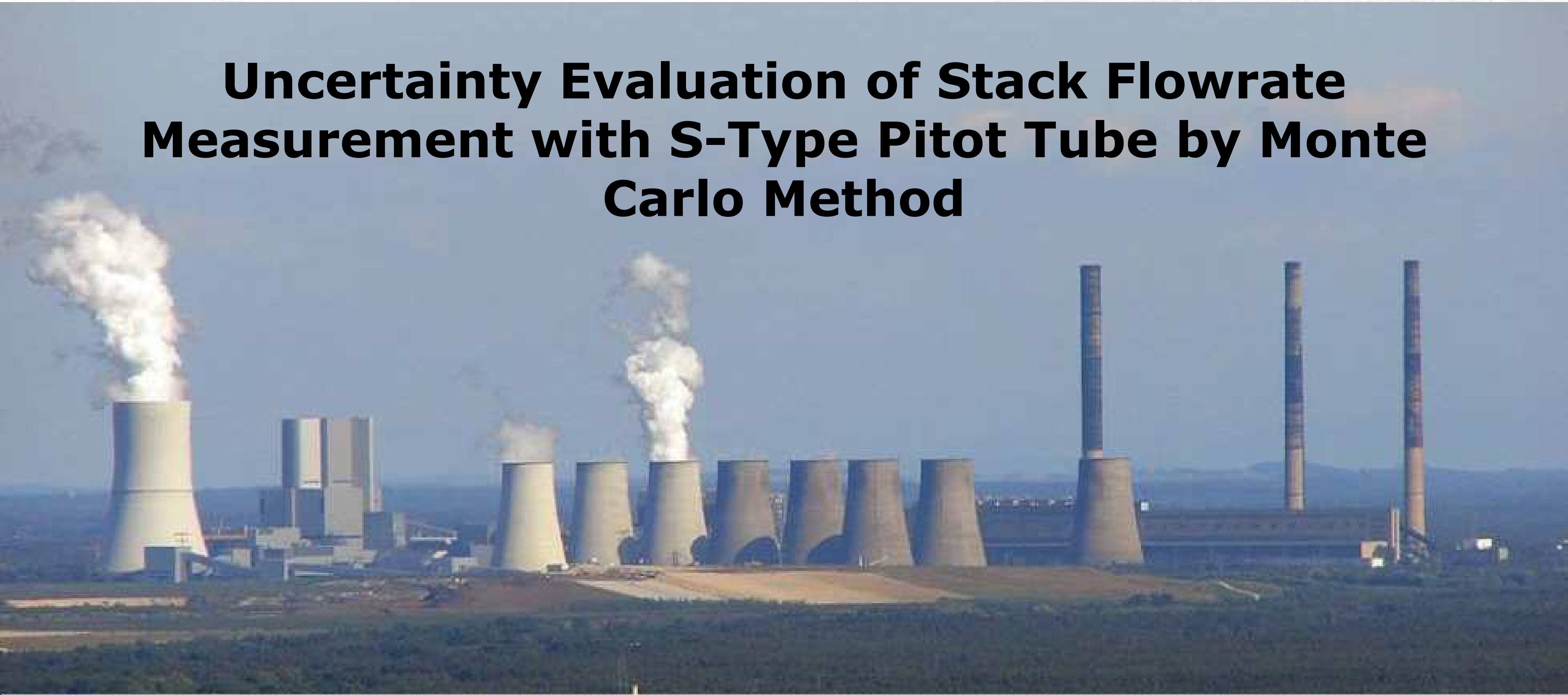
Uncertainty Evaluation of Stack Flowrate Measurement with S-Type Pitot Tube by Monte Carlo Method



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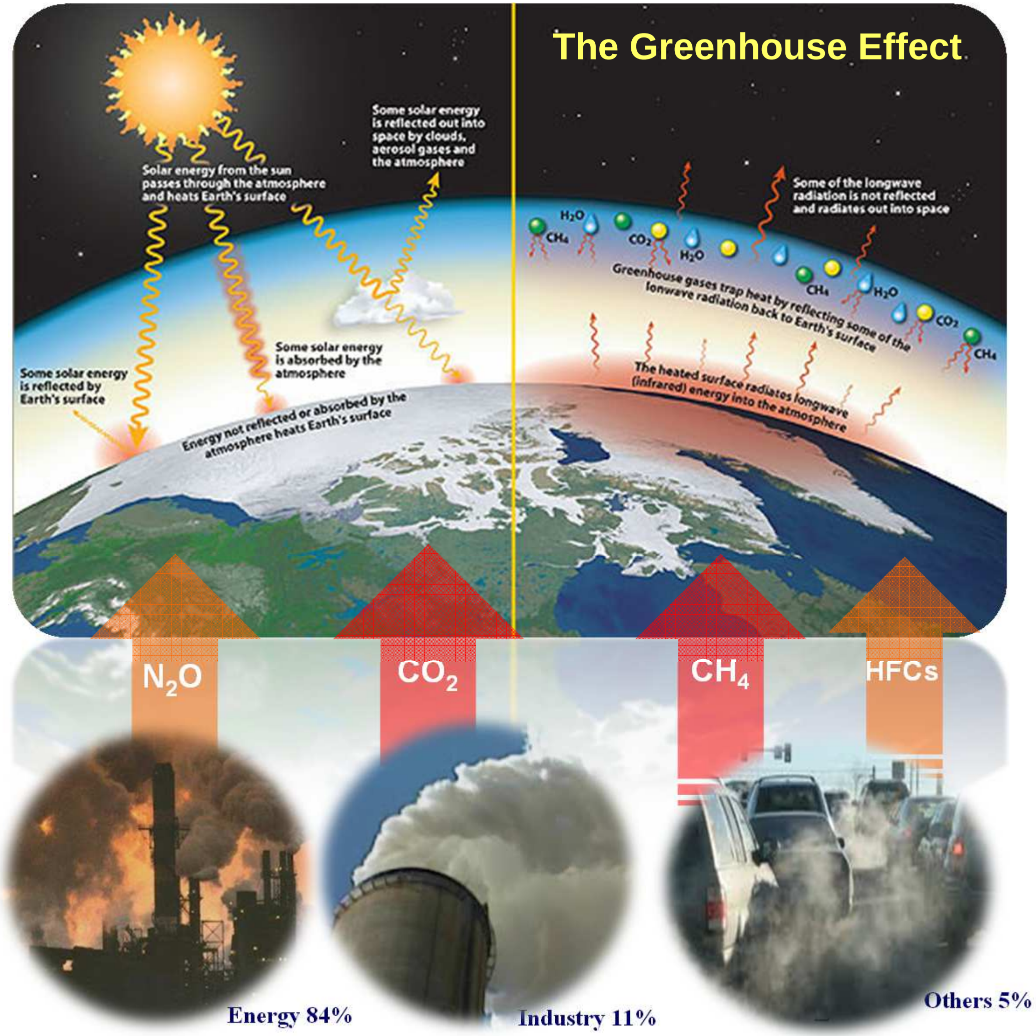
Contents

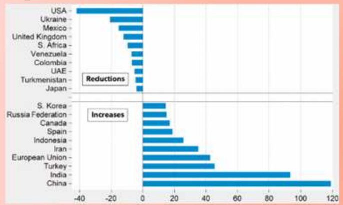
Greenhouse Gas Emission



Energy and Industry → 95% of GHGs emission in KOREA

UNFCCC(UN Framework Convention on Climate Change), 1994
Kyoto Protocol, 1997

Top 10 countries increase GHGs 2017



(BP Statistical review of World energy June 2018)

"30% reduction of GHGs until 2020"
KOREA GHG Emission Trading Scheme Since Jan 2015

Needs of accurate and reliable measurement and estimation GHGs emissions

Proper Uncertainty Analysis

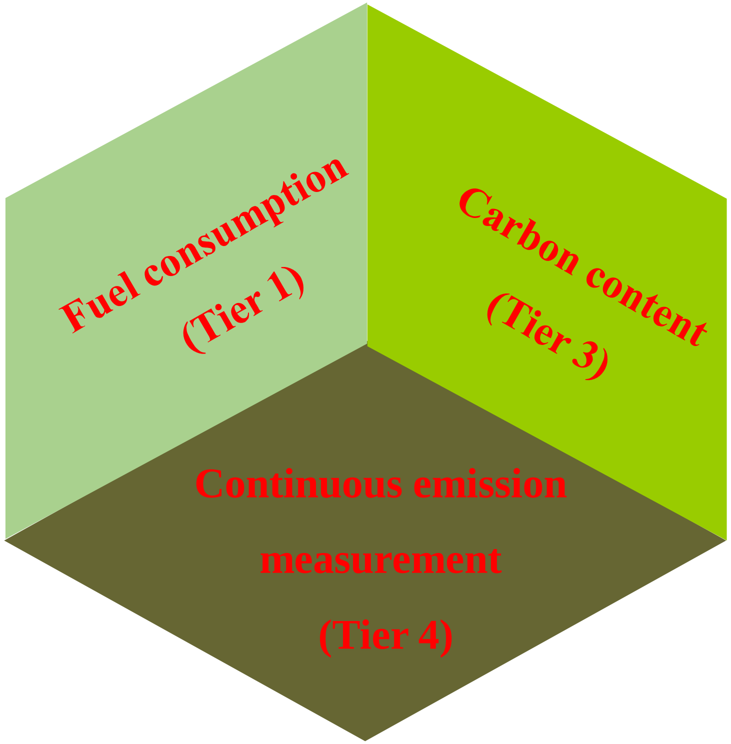
Methodology for estimating GHG emission

Fuel consumption (Tier 1)

by IPCC guidelines, EPA

$$E = FC \times EF = FM \times NCV \times EF$$

E : estimated emission (kg)
FC : fuel consumption (TJ)
EF : emission factor (kg/TJ)
FM : fuel mass (kg)
NCV : net calorific value(TJ/kg)



Carbon content (Tier 3)

by IPCC guidelines, EPA

$$E = FM \times CC \times \frac{M_{CO_2}}{M_C}$$

CC : fuel carbon content(kg/kg)
M_{CO2} : molecular mass of carbon dioxide
M_C : atomic mass of carbon

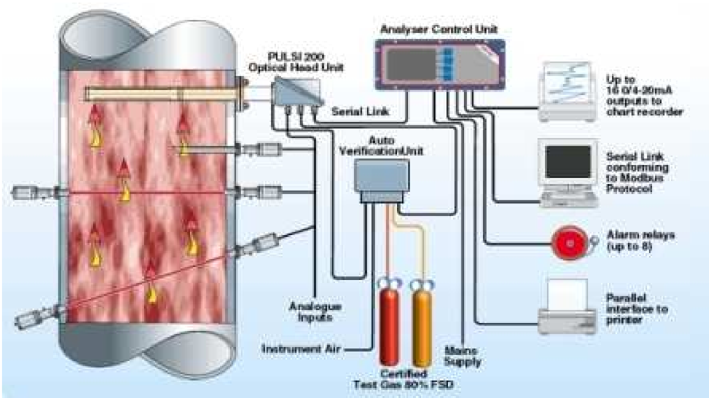
Methodology for estimating GHG emission

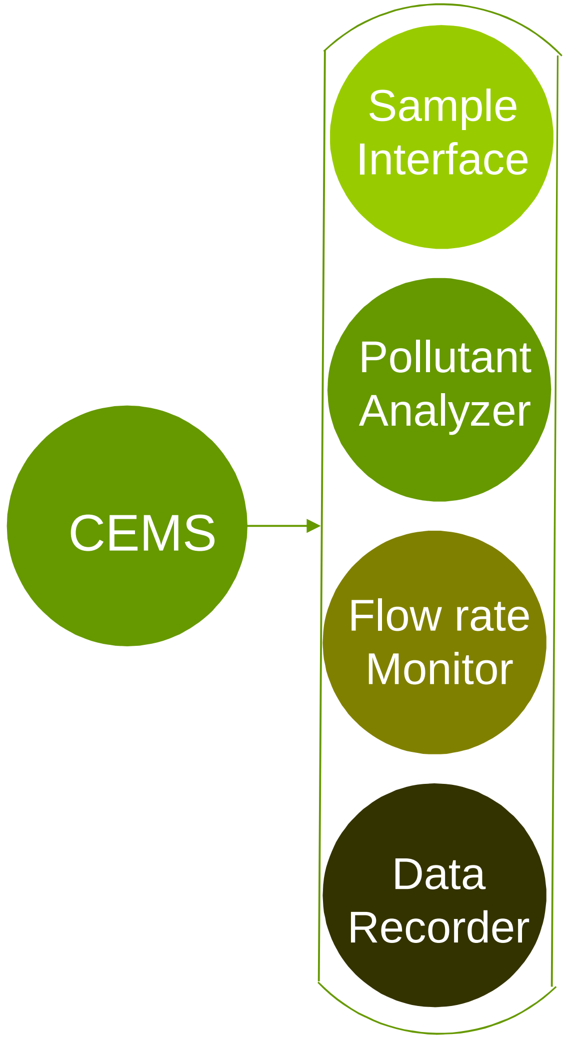
Continuous emission measurement (Tier 4)

by IPCC guidelines, EPA

$$E = \sum_{i=1}^{N} E_{5min,i} = \sum_{i=1}^{N} (x_{5min,i} \times Q_{5min,i} \times \frac{M_{gas}}{MV})$$

$E_{5min,i}$: 5-min accumulated emission of *i*th measurement (kg)
$x_{5min,i}$: 5-min averaged concentration of the ith measurement(% or ppm)
$Q_{5min,i}$: 5-min accumulated volumetric flow of the *i*th measurement (m^3)
M_{gas} : molar mass of an emission gas, *MV* is the molar volume of ideal gas
N : total number of 5-min estimated emissions.





Measurement of the flow rate in stacks

- **Methods**

U.S. EPA Method 2: Determination of Stack Gas Velocity and Volumetric Flow Rate

U.S. EPA Method 4: Determination of Moisture Content in Stack Gases

Korea Ministry of Environment ES. 01809.1: Test Method on air pollution

- **Equation for The 5-min accumulated volumetric flow rate**

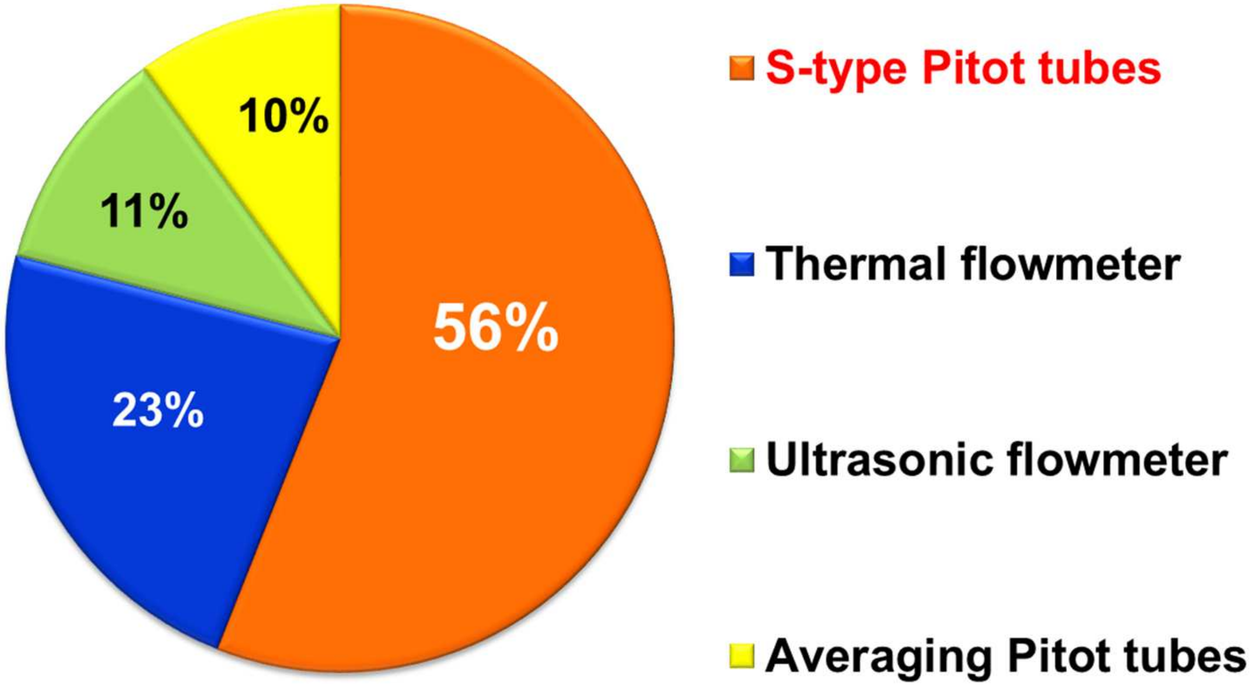
$$Q_{5min} = \bar{V} \times \frac{\pi D^2}{4} \times \frac{P_s}{760} \times \frac{273.15}{T_s} \times (1 - x_w) \times \mathrm{t}$$

Q_{5min} : dry volumetric flowrate at stack (m^3/min)
$\bar{V}$: average velocity (m/min)
D : diameter of the stack
T_s : average temperature of the stack gas (K)
P_s : average absolute pressure of the stack gas (mmHg)
X_w : water content of the stack gas (%)

Estimating Uncertainties of Stack Gas Flow rate measurement for CEM by GUM and MCM

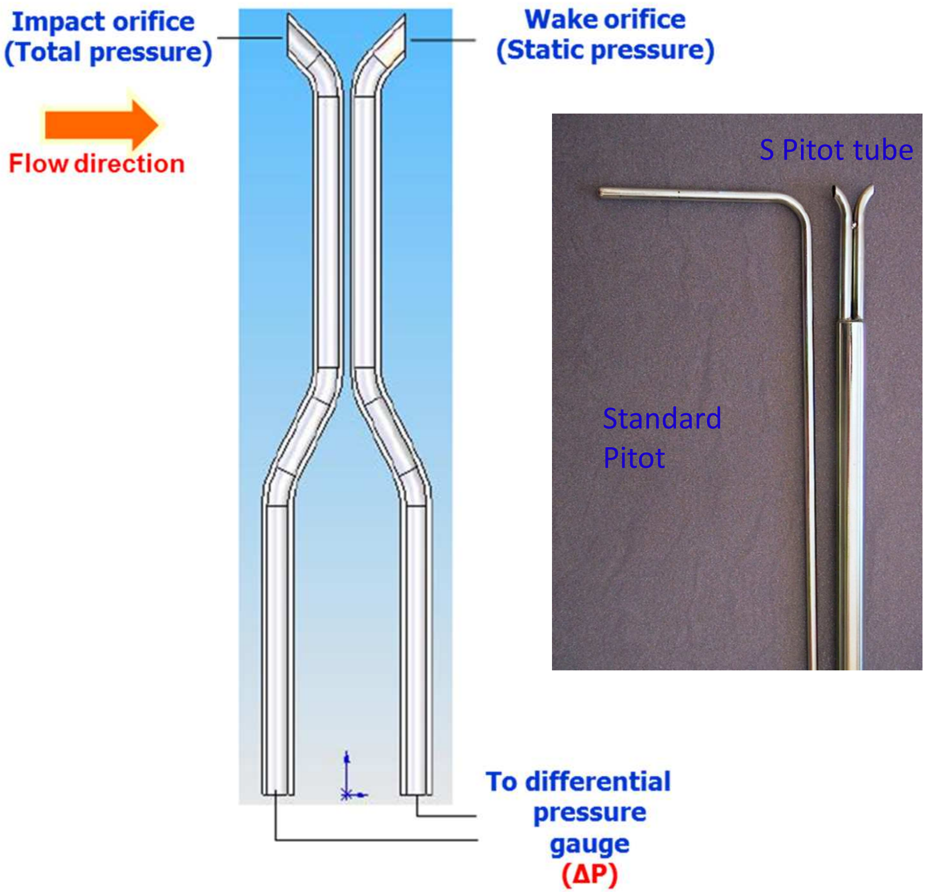
On-site measurement with S-type Pitot

Instruments for measuring velocity in stacks in Korea



On-site measurement with S-type Pitot

S-type Pitot tube



Characteristics of S-type Pitot

- Structure follows: ISO 10780, KS M9429, EPA Title 40: Part 60, Appendix A method2
- Large pressure orifices(Φ=5~10mm) & Strong tubes for high dust environments
- Measurement differential pressure between an impact and wake orifice based on Bernoulli equation

$$V_s = C_P \times \sqrt{\frac{2\Delta P}{\rho}}$$

V_s: flow velocity in the stack gas(*m/s*)

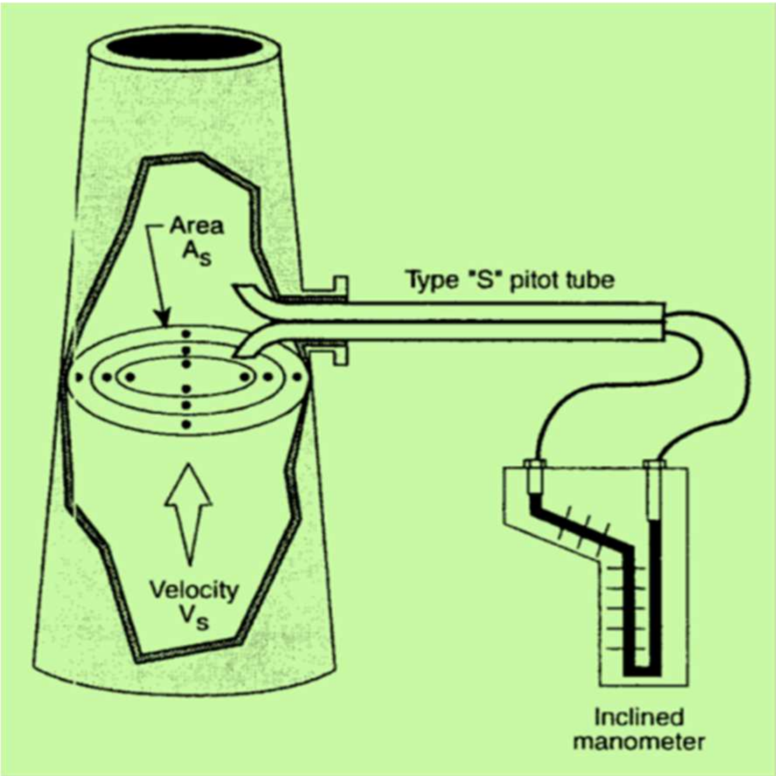
C_P : S type Pitot tube coefficient

ΔP : differential pressure between impact and wake orifice (*Pa*)

ρ : density of the stack gas (kg/m^3)

On-site measurement with S-type Pitot

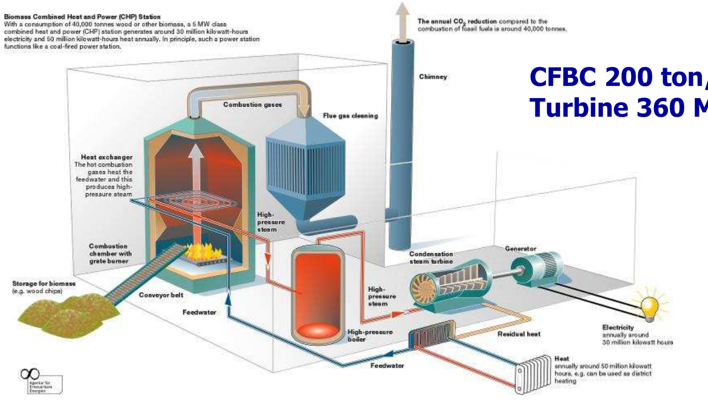
- Sampling traverse points in the stack for velocity distribution by ISO 10780 / EPA method 1



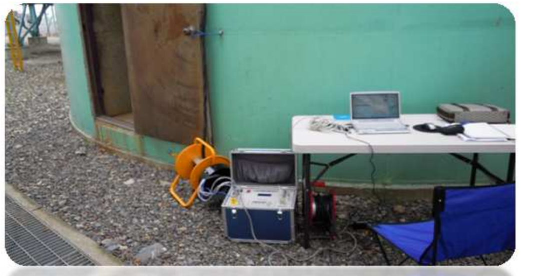
Stack Diameter 2R (m)	raidus	numbers	Distance from center of stack				
			r_1	r_2	r_3	r_4	r_5
< 1	1	4	0.707 R	-	-	-	-
1 ~ 2	2	8	0.500 R	0.866 R	-	-	-
2 ~ 4	3	12	0.408 R	0.707 R	0.913 R	-	-
4 ~ 4.5	4	16	0.354 R	0.612 R	0.791 R	0.935 R	-
> 4.5	5	20	0.316 R	0.548 R	0.707 R	0.837 R	0.949 R

On-site measurement with S-type Pitot

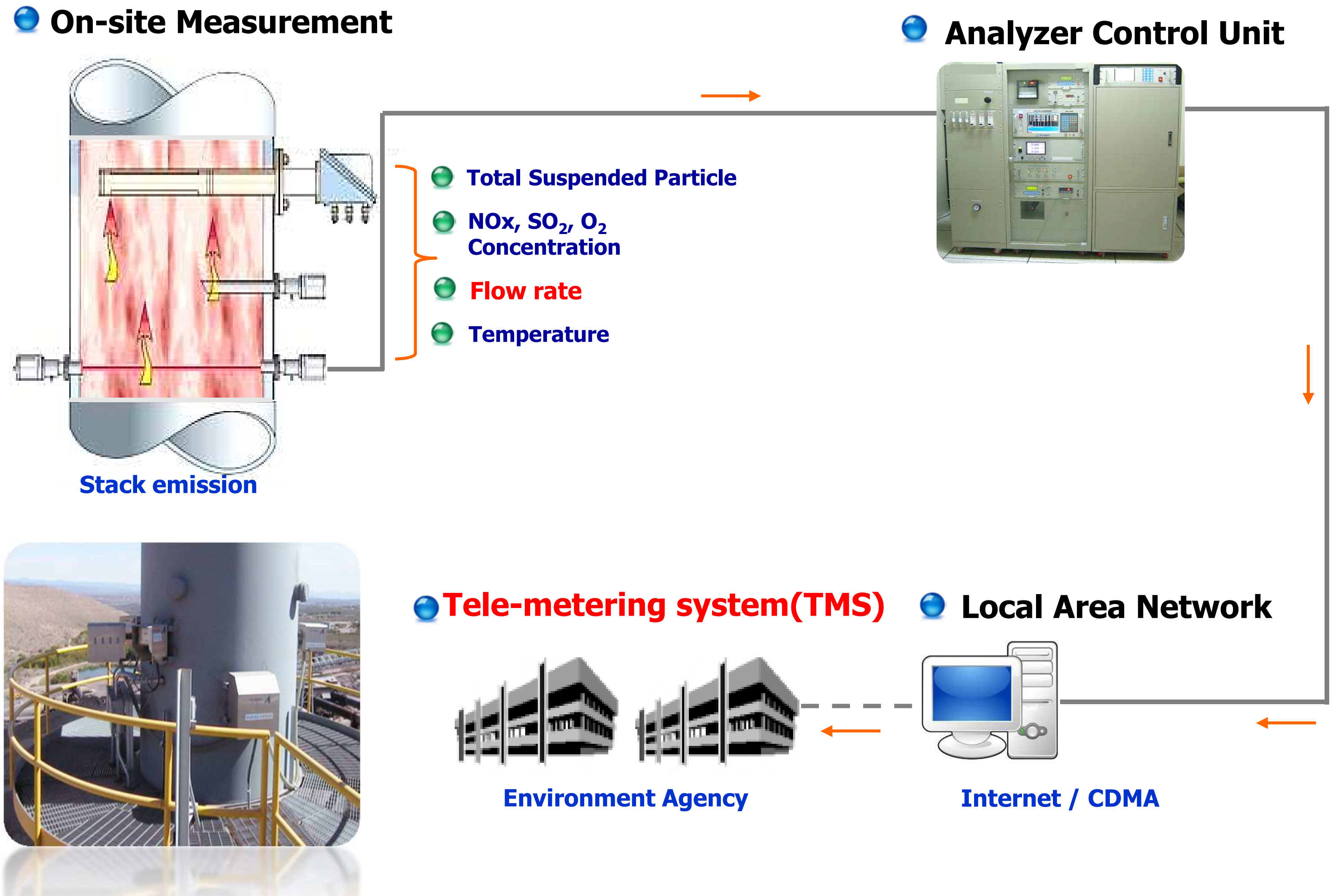
- Combined heat and power plan at Gunjang Energy Co., Ltd



CFBC 200 ton/hr
Turbine 360 MW



On-site measurement with S-type Pitot



Uncertainty evaluation

Modelling for dry volumetric in the stack

$$Q_{5min} = C_P \times \sqrt{\frac{2\Delta P}{\rho}} \times \frac{\pi D^2}{4} \times \frac{P_s}{760} \times \frac{273.15}{T} \times (1 - x_w) \times 300$$

GUM method

- **ISO/IEC Guide 98-3: Guide to the Expression of Uncertainty in Measurement**
- **Law of Propagation of Uncertainty through Taylor series approximation**

$$u_c^2(Q) = c_{C_p}^2 u^2(C_p) + c_{\Delta P}^2 u^2(\Delta P) + c_\rho^2 u^2(\rho) + c_D^2 u^2(D)$$
$$+ c_{T_s}^2 u^2(T_s) + c_{P_s}^2 u^2(P_s) + c_{(1-X_w)}^2 u^2(1 - X_w)$$

Uncertainty evaluation by GUM

Sensitivity coefficient

$$C_{C_p} = \frac{\partial Q}{\partial C_p} = \times\sqrt{\frac{2\Delta P}{\rho}} \times \frac{\pi}{4}D^2 \times \frac{P_s}{P_{std}} \times \frac{T_{std}}{T_s} \times (1 - X_w)$$

$$C_{\Delta P} = \frac{\partial Q}{\partial \Delta P} = \frac{1}{2}C_P \times \sqrt{\frac{2}{\rho \Delta P}} \times \frac{\pi}{4}D^2 \times \frac{P_s}{P_{std}} \times \frac{T_{std}}{T_s} \times (1 - X_w)$$

$$C_\rho = \frac{\partial Q}{\partial \rho} = \frac{1}{2}C_P \times \sqrt{\frac{2\Delta P}{\rho^3}} \times \frac{\pi}{4}D^2 \times \frac{P_s}{P_{std}} \times \frac{T_{std}}{T_s} \times (1 - X_w)$$

$$C_D = \frac{\partial Q}{\partial D} = C_P \times \sqrt{\frac{2\Delta P}{\rho}} \times \frac{\pi}{2}D \times \frac{P_s}{P_{std}} \times \frac{T_{std}}{T_s} \times (1 - X_w)$$

$$C_{P_s} = \frac{\partial Q}{\partial P_s} = C_P \times \sqrt{\frac{2\Delta P}{\rho}} \times \frac{\pi}{4}D^2 \times \frac{1}{P_{std}} \times \frac{T_{std}}{T_s} \times (1 - X_w)$$

$$C_{T_s} = \frac{\partial Q}{\partial T_s} = -C_P \times \sqrt{\frac{2\Delta P}{\rho}} \times \frac{\pi}{4}D^2 \times \frac{P}{P_{std}} \times \frac{T_{std}}{T_s^2} \times (1 - X_w)$$

$$C_{(1-X_w)} = \frac{\partial Q}{\partial (1 - X_w)} = -C_P \times \sqrt{\frac{2\Delta P}{\rho}} \times \frac{\pi}{4}D^2 \times \frac{P}{P_{std}} \times \frac{T_{std}}{T_s^2}$$

Relative uncertainty

$$\frac{u_c^2(Q)}{Q} = \frac{u^2(C_p)}{C_P^2} + \frac{1}{4}\frac{u^2(\Delta P)}{\Delta P^2} + \frac{1}{4}\frac{u^2(\rho)}{\rho^2} + 4\frac{u^2(D)}{D^2} + \frac{u^2(P_s)}{P_s^2} + \frac{u^2(T_s)}{T_s^2} + \frac{u^2(1 - X_w)}{(1 - X_w)^2}$$

Uncertainty evaluation by GUM

S-type Pitot Coefficient (C_p)

$$\frac{u^2(C_p)}{C_P^2}$$

- **S-type Pitot tube coefficient and uncertainty are determined by calibration certification**
- **S-type Pitot tube was calibrated in the wind tunnel of the accredited calibration laboratories**

  - Korea Environment Corporation (U = 1.1 %, k = 2 with 95% confidence level)

Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient c_i	Uncertainty Contribution to u(Q) $u_i \times c_i$ (%)
Type A	0.826	N/A	-	1	-
Type B		0.55%	Normal	1	0.55 %

Uncertainty evaluation by GUM

Different pressure (ΔP)

$$\frac{u^2(\Delta P)}{\Delta P}$$

- **Type A - collective data every 3 seconds for 5 minutes**
- **Type B – annual variation of linearity results in the performance test between 2009 & 2010**

Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution $u_i \times c_i$ (%)
Type A	136.4 Pa	0.54%	Normal	1/2	0.27 %
Type B		1.78 %	Rectangular	1/2	0.89 %

Uncertainty evaluation by GUM

Gas density (ρ) $\frac{u^2(\rho)}{\rho}$

- **Weighted average based on concentration of major gas components (N_2, CO_2, O_2, Ar, Water)**

$$\rho_0 = \frac{(\%CO_2 \times 44 + \%O_2 \times 32 + \%Ar \times 39.94 + \%N_2 \times 28 + \%X_w \times 18) \times 100}{22.4}$$

$$\rho_{std} = \rho_0 \times \frac{T_{std}}{T_s} \times \frac{P_s}{P_{std}}$$

- **Type A - collective data every 10 seconds for 5 minutes**
- **Type B - difference between calculating gas density value and used theoretical value (1.3 kg/m^3 by test method in environment ministry**

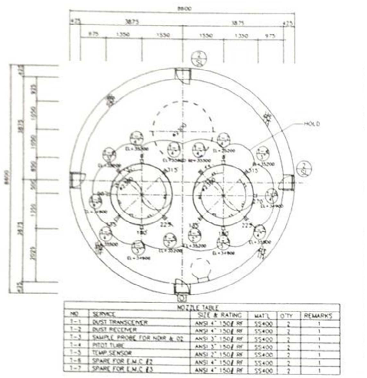
Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution u_i x c_i (%)
Type A	1.33 kg/m^3	0.0054 %	Normal	1/2	0.0027 %
Type B		1.12 %	Rectangular	1/2	0.61 %

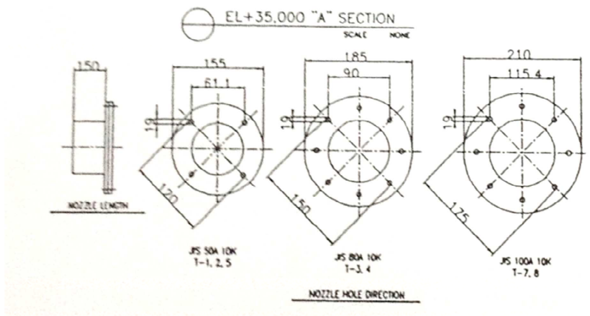
Uncertainty evaluation by GUM

Stack diameter (D)

$$\frac{u^2(D)}{D}$$

- **The manufacture's technical specification with the value 2500 mm**
- **The resolution of tape measure tool ± 10 mm**





Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution $u_i \times c_i$ (%)
Type A	2500 mm	N/A	-	2	-
Type B		0.23 %	Rectangular	2	0.46%

Uncertainty evaluation by GUM

Static pressure (P_S)

$$\frac{u^2(P_s)}{P_s}$$

- **Type A - collective data every 3 seconds for 5 minutes**
- **Type B - below 1 mmHg from calibration certificates**

Temperature (T_S)

$$\frac{u^2(T_s)}{T_s}$$

- **Type A - collective data every 3 seconds for 5 minutes**
- **Type B - below 1 K from calibration certificates**

Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution $u_i \times c_i$ (%)
Type A	756 mmHg	0.0019 %	Normal	1	0.0019 %
Type B		0.13 %	Rectangular	1	0.15 %

Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution $u_i \times c_i$ (%)
Type A	409 K	0.0048 %	Normal	1	0.0048 %
Type B		0.14 %	Rectangular	1	0.14 %

Uncertainty evaluation by GUM

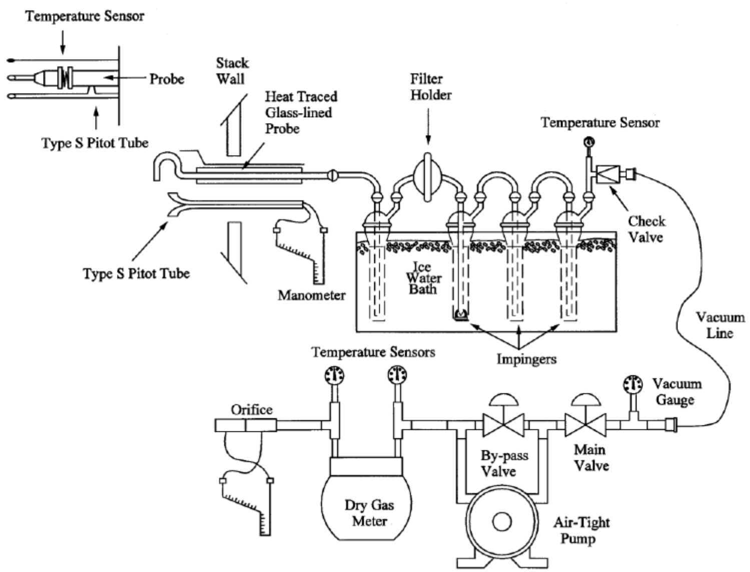
Water content (X_w)

$$\frac{u^2(1-X_w)}{(1-X_w)}$$

- **EPA method 4 (Determination of moisture content in stack), Test method in Korea Environ.**
- **Water content is calculated by condensed moisture in the impinger and volume flow rate in the dry gas meter**

$$X_w = \frac{\frac{m_a}{18} + \left(\frac{P_w}{P_{std}} \times V_m \times \frac{T_{std}}{T_m} \times \frac{P_m}{P_{std}}\right) \times \frac{1}{22.4}}{\frac{V_m \times \frac{T_{std}}{T_m} \times \frac{P_m}{P_{std}}}{22.4} + \frac{m_a}{18}}$$

m_a : increment mass of water, (g/min)
P_w : water vapor pressure (mmHg)
V_m : dry gas volume by gas meter (L/min)
P_m : pressure at gas meter (mmHg)
T_m : temperature at gas meter (K)



Uncertainty evaluation by GUM

- **Type A - continuous moisture mass measurement method (0.292 g/min) in every 10 seconds**
- **Type B - difference between measured water content value and used theoretical value (8.1%)**

  - estimated by numerical derivative method (GUM) in water content equation

Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution $u_i \times c_i$ (%)
Type A	91. 5%	0.0016 %	Normal	1	0.0016 %
Type B		0.30 %	Rectangular	1	0.30 %

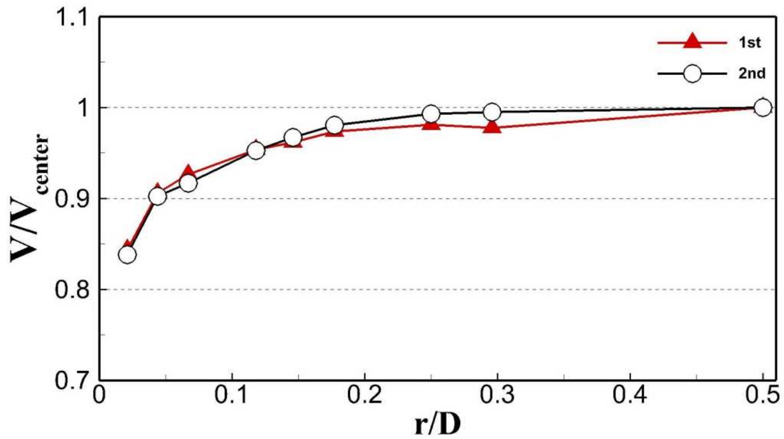
Uncertainty evaluation by GUM

Velocity distribution (ΔV)

$u(\Delta V)$

- **Velocities at 10 sampling traverse points were measured according to EPA method 1& Test method**
- **S-type Pitot tube in the stack is typically fixed in a certain position**
- **Type B - Deviation of averaged velocity by velocity distribution with velocity at fixed position in the cross section of stack**

Uncertainty Component	Value	Standard uncertainty u_i (%)	Probability distribution	Sensitivity Coefficient	Uncertainty Contribution u_i (%)
Type A	14.8 m/s				
Type B		1.54 %		1	1.54 %



Uncertainty evaluation by GUM

Uncertainty Budget

Symbol	Value	unit	Uncertainty component		Sensitivity coefficient	Combined uncertainty contribution
			Type A %	Type B %		
C_p	0.826	-	-	0.55	1	0.55 %
ΔP	136.4	Pa	0.54	1.78	0.5	0.93 %
ρ	1.33	kg/m³	0.0054	1.12	0.5	0.56 %
D	2500	mm	-	0.23	2	0.46 %
P_s	756	mmHg	0.0019	0.15	1	0.15 %
T_s	409	K	0.0048	0.16	1	0.16 %
$1-X_w$	91.5	%	0.0016	0.30	1	0.30 %
ΔV_D	14.8	m/s	1.54	-	1	1.54 %
Q	12972.5	m³/min (5min)				
Combined uncertainty of the flow rate measurement						2.05 %
95 % confidence level, k=						2
Expanded Uncertainty, U =						4.1 %

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Uncertainty evaluation by MCM

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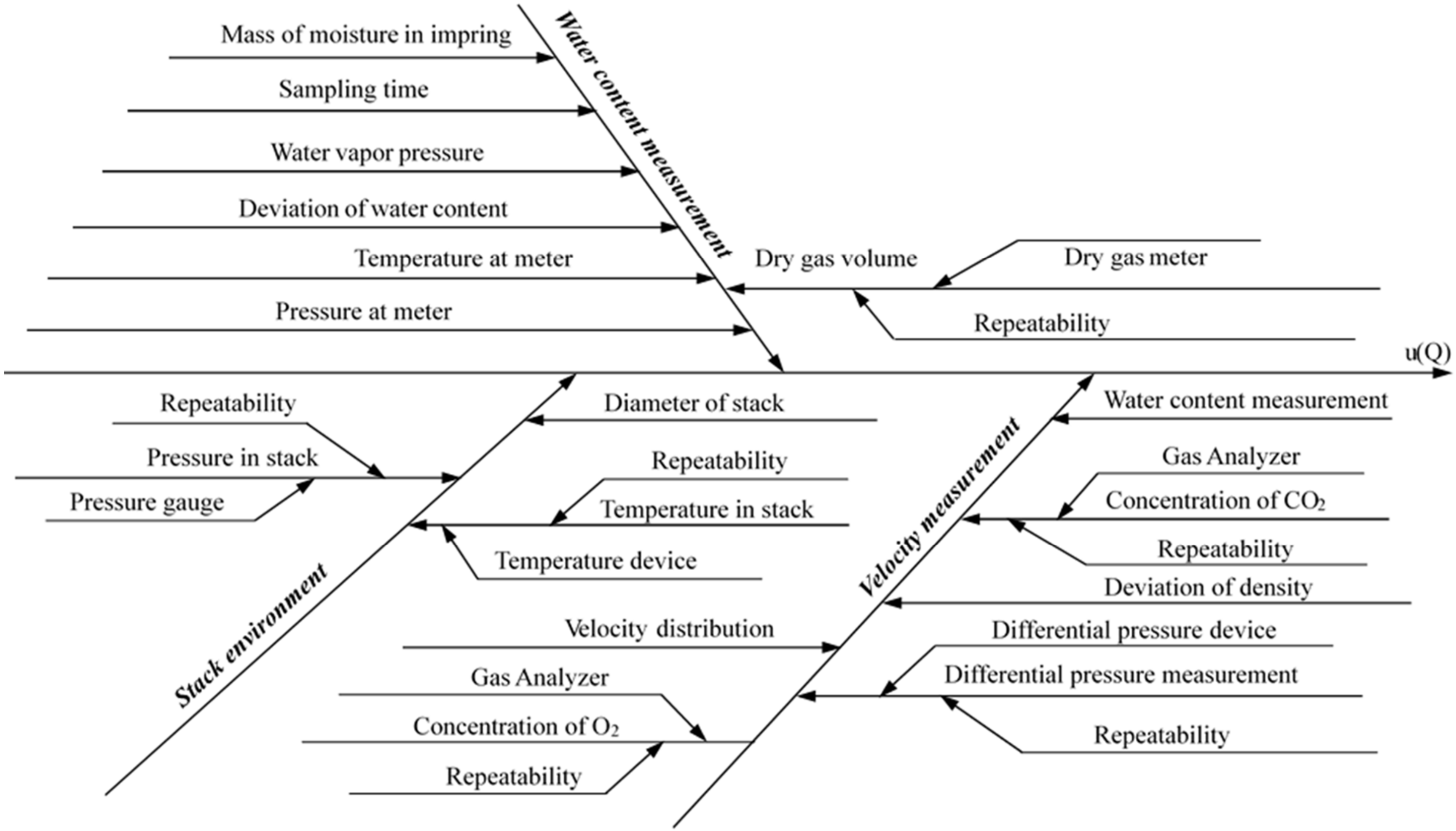
Sources of measurement uncertainty contributions



Uncertainty evaluation by MCM

Sources of measurement uncertainty contributions

- **Propagation of a joint probability distribution**
- **Outcome is a set of trials from the probability distribution associated with the measurand**

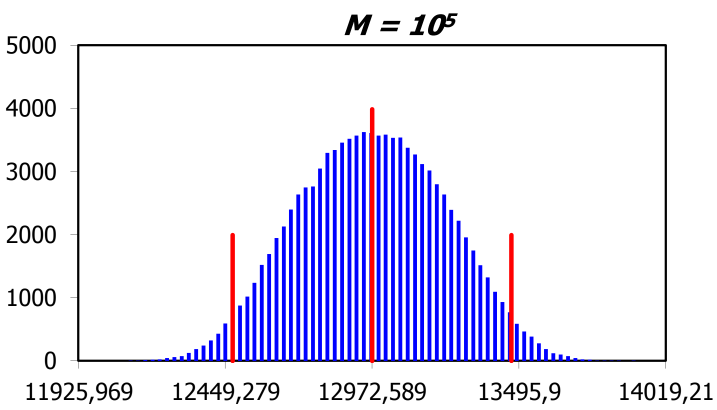
$$X_1^{(m)} \sim g_{X_1}$$
$$X_2^{(m)} \sim g_{X_2}$$
$$\vdots$$
$$X_N^{(m)} \sim g_{X_N}$$
$$\rightarrow Y = f(X_1, X_2, \dots X_N)$$
$$\downarrow$$
$$\{Y^{(m)} | m = 1 \dots M\} \approx g_Y$$

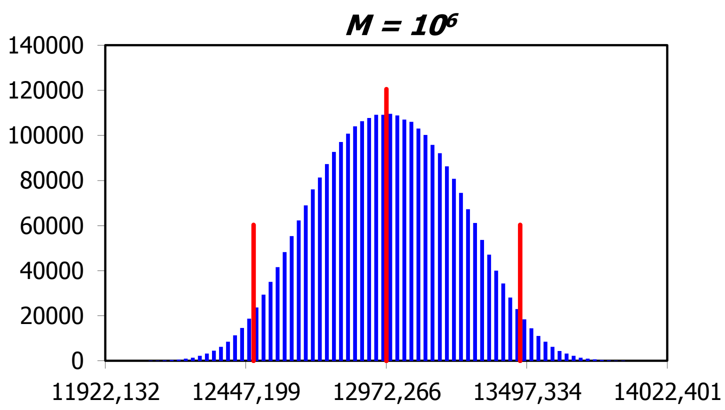
Parameter input	Magnitude	Unit	Distribution
Concentration of CO_2, $\%CO_2$	$X_1 = 15.24$	%	t-distribution
Gas analyzer for CO_2	$X_2 = 0$	%	Gauss
Concentration of O_2, $\%O_2$	$X_3 = 4.028$	%	t-distribution
Gas analyzer for O_2	$X_4 = 0$	%	Gauss
Dry gas volume, Q_m	$X_5 = 4.61$	L/min	t-distribution
Dry gas meter	$X_6 = 0$	L/min	Gauss
Mass of moisture, m_0	$X_7 = 16.456$	g	Rectangular
Mass of moisture, m_{5min}	$X_8 = 17.981$	g	Rectangular
Sampling time, t_0	$X_9 = 0.1$	min	Rectangular
Sampling time, t_{5min}	$X_{10} = 0$	min	Rectangular
Pressure in stack, P_s	$X_{11} = 756$	mmHg	t-distribution
Pressure gauge	$X_{12} = 0$	mmHg	Rectangular
Temperature in stack, T_s	$X_{13} = 409$	K	t-distribution
Temperature device	$X_{14} = 0$	K	Rectangular
Deviation of density, $\Delta\rho$	$X_{15} = 0$	kg/m^3	Rectangular
Different pressure, ΔP	$X_{16} = 136.4$	Pa	Gauss
Different pressure gauge	$X_{17} = 0$	Pa	Gauss
Temperature at meter, T_m	$X_{18} = 297.75$	K	Rectangular
Pressure at meter, P_m	$X_{19} = 771$	mmHg	Rectangular
Water vapor pressure, P_w	$X_{20} = 5.69$	mmHg	Rectangular
Calibration of S-type Pitot, C_P	$X_{21} = 0.826$		Gauss
Diameter of stack, D	$X_{22} = 2.5$	m	Rectangular
Velocity distribution, ΔV	$X_{23} = 0$	m/s	Rectangular
Deviation of moisture, ΔX_w	$X_{24} = 0$	%	Rectangular

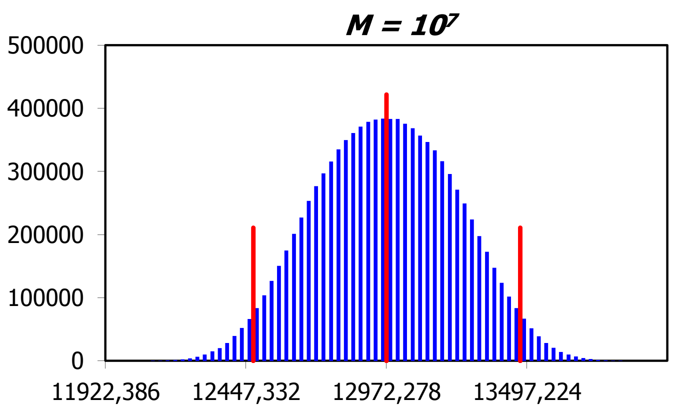
Uncertainty evaluation by MCM

Number of Monte Carlo runs

- **The Monte Carlo simulation process carried by number of Monte Carlo trials (M)**
- **M number decides shape of probability density function for the dry flow rate**
- **In this study the number of model evaluations simulated respectively M = 10^5, 10^6, and 10^7**







Monte Carlo simulation

Framework of this simulation was based on the Microsoft Excel and Microsoft Visual Basic for

Mathematical model	FXN = 300 * (X(21) * SQR(2 * (X(15) + X(16) + X(17)) / ((((44 * (X(1) + X(2)) + 32 * (X(3) + X(4)) + 39.94 * 0.93 + 18 * (Xw + X(24)) + 28 * (99.06 - (X(1) + X(2)) - (X(3) + X(4)) - (Xw +
Number of quanlity	24
Bins	120
Decimal places	5
Status	Finish after 62.54 seconds at 오후 9:58:07 in 2019-

Monte Carlo

Confidence level	95 %
Trials	3,000,000

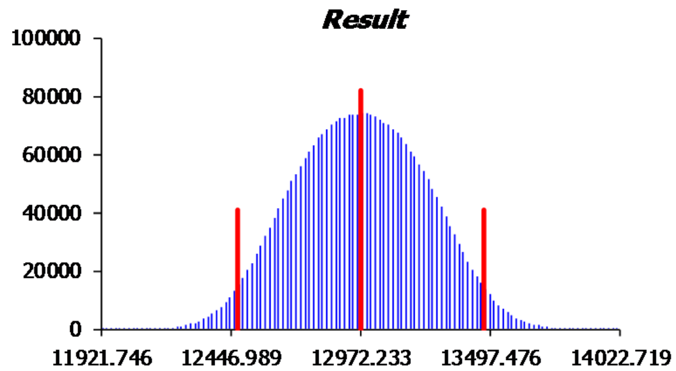
☑ Draw histogram after sampling

☐ Create image of drawing of histogram

№	Quanlity	Symbol	Value	Distribution	Code	Factor 1	Factor 2	Divisor	u_i	c_i	$(u_i \cdot c_i)^2$	%	
01	$\%CO_2$	x_1	15.2407	t-distribution	4	0.04497	27	1	0.00865	-34.758	9.0E-02	0.00	OK
02	Gas analyzer	x_2	0	Gaussian	0	0.0762037		1	0.0762	-34.747	7.0E+00	0.01	OK
03	$\%O_2$	x_3	4.02778	t-distribution	4	0.049562	27	1	0.00954	-8.6895	6.9E-03	0.00	OK
04	Gas analyzer	x_4	0	Gaussian	0	0.0201389		1	0.02014	-8.6893	3.1E-02	0.00	OK
05	V_m	x_5	4.61407	t-distribution	4	0.00501	27	1	0.00096	185.977	3.2E-02	0.00	OK
06	Device for V_m	x_6	0	Gaussian	0	0.02399316		1	0.02399	185.079	2.0E+01	0.03	OK
07	Mass m_0	x_7	16.456	Rectangular	1	-0.001	0.001	1.73205	0.00058	586.944	1.1E-01	0.00	OK
08	Mass m_1	x_8	17.918	Rectangular	1	-0.001	0.001	1.73205	0.00058	-586.94	1.1E-01	0.00	OK
09	Time t_0	x_9	0.1	Rectangular	1	-0.0166667	0.01666667	1.73205	0.00962	-171.62	2.7E+00	0.00	OK
10	Time t_1	x_{10}	5.1	Rectangular	1	-0.0166667	0.01666667	1.73205	0.00962	171.622	2.7E+00	0.00	OK
11	Pressure P_s	x_{11}	756.342	t-distribution	4	0.07338	100	1	0.00734	8.57577	4.0E-03	0.00	OK
12	Device for P_s	x_{12}	0	Rectangular	1	-1	1	1.73205	0.57735	8.57294	2.4E+01	0.04	OK
13	Temperature T_s	x_{13}	409.24	t-distribution	4	0.03415	100	1	0.00342	-15.849	2.9E-03	0.00	OK
14	Device for T_s	x_{14}	0	Rectangular	1	-1	1	1.73205	0.57735	-15.82	8.3E+01	0.12	OK
15	Deviation of density	x_{15}	0	Rectangular	1	0.016466	-0.016446	1.73205	0.00951	-7178	4.7E+03	6.79	OK
16	Device for ΔP	x_{16}	136.31	Gaussian	0	0.542806		1	0.54281	47.5843	6.7E+02	0.97	OK
17	Stability of device for ΔP	x_{17}	0	Gaussian	0	2.54422615		1	2.54423	47.3643	1.5E+04	21.16	OK
18	Temperature T_m	x_{18}	297.75	Rectangular	1	-1	1	1.73205	0.57735	-2.882	2.8E+00	0.00	OK
19	Pressure P_m	x_{19}	771	Rectangular	1	-1	1	1.73205	0.57735	1.11298	4.1E-01	0.00	OK
20	Water Vapor pressure P_w	x_{20}	5.69	Rectangular	1	-1	1	1.73205	0.57735	-14.563	7.1E+01	0.10	OK
21	Coefficient of S Pitot C_p	x_{21}	0.826	Gaussian	0	0.004543		1	0.00454	15705.1	5.1E+03	7.42	OK
22	Diameter D	x_{22}	2.5	Rectangular	1	-0.01	0.01	1.73205	0.00577	10377.9	3.6E+03	5.23	OK
23	Velocity distribution	x_{23}	0	Rectangular	1	0.3865	-0.3865	1.73205	0.22315	895.015	4.0E+04	58.12	OK
24	Deviation of moisture	x_{24}	0	Rectangular	1	-0.002	0.002	1.73205	0.00115	0	0.0E+00	0.00	OK

GUM value	12972.42814
GUM uncertainty	523.93879
MC value	12972.23267
MC uncertainty	525.24319
MC lower	-497.08643
MC upper	499.59165

Value of axis Y in percent



Conclusion

1. Uncertainty Evaluation Result

- **By GUM method:**
  Q(every 5 minutes) = 12972.7 m^3
  U(Q) = 4.1 % with k = 2, P = 95 %

- **By Monte Carlo method:**
  Q(every 5 minutes) = 12972.2 m^3
  U(Q) = 4.0% (525.2 m^3) with k = 2, P = 95 %

- Agreement between two methods
- Reliable to use the Monte Carlo method in a complicate mathematical model

2. The main components affect the dry flow rate measurement

- $\Delta V, \Delta P, \Delta\rho, C_P$: contribute dominantly to the uncertainty of the dry flow rate measurement

Thank you for your kind attention!